Human Induced Floor Vibrations
Technical Background
AISC Design Guide 11
Part 1 / 2

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1. Introduction

- Problem Statement
  - Human activity cause the structure to vibrate.
  - Occupants might be irritated or frightened.
  - Sensitive equipment might malfunction.
  - Designer must be able to evaluate vibration serviceability during design.

- Serviceability limit state.
  - Not a strength or fatigue issue.
  - Panic possible in rare and very extreme cases.
1. Introduction

- A very realistic limit state
  - Steel-Framed Structures: floor vibration is about as likely to control as strength and deflection limit states.
- These are especially vulnerable:
  - Areas with rhythmic activities.
  - Areas with sensitive equipment.
  - Monumental stairs.
1. Introduction

- Why focus on human induced floor vibration?
  - Vibration Due to Equipment
    - Can usually be isolated at the source.
    - High frequency → more difficult to feel.
    - May still be an issue in some cases.
  - Wind induced lateral vibration. A different subject.
  - Earthquake design has to do with safety. A different subject.
1. Introduction

- Floor Vibration Evaluation in a Nutshell
  - Compare predicted response to the tolerance limit → OK or No Good

- Predict Response
  - Forcing function.
  - Dynamic properties of the structure.
  - Response prediction method.

- Define Tolerance Limit
  - Human annoyance.
  - Equipment problems.
1. Introduction

- This Presentation
  1. Introduction
  2. Technical background
  3. DG11 Chapter 4
    1. Derivation and Implementation
    2. Example
    3. Accuracy Investigation
  4. DG11 Chapter 5
  5. Remedial Options Case Studies

Part 2
2. Technical Background: Dynamic Forces

- When humans walk, run, bounce, sway, or jump, inertial forces cause dynamic loads.

- Example:

![Diagram showing ground reaction forces during walking.](image)

Average Walking (120 bpm) Footstep
2. Technical Background: Dynamic Forces

- Footstep forces are complicated.

![Graph showing ground reaction forces over time](graph.png)

- Need simple mathematical representations
- Fourier Series
- Individual Footstep Pulse or Impulse
2. Technical Background: Dynamic Forces

- Fourier Series vs Individual Footstep
  - Fourier series useful for computing resonant response.
  - Pulse or impulse useful for computing response to one footstep.
Review of Fourier Series

Any periodic function is a superposition of the average plus an infinite number of sinusoids.

Each sinusoid (called a harmonic) has a different amplitude \( (Amp_h) \), frequency \( (f_h) \), and phase lag \( (\phi_h) \).

\[
F(t) = \text{Average Value} + \sum_{h=1}^{\infty} Amp_h \cdot \sin(2 \cdot \pi \cdot f_h \cdot t - \phi_h)
\]

\[
\approx \text{Average Value} + \sum_{h=1}^{N} Amp_h \cdot \sin(2 \cdot \pi \cdot f_h \cdot t - \phi_h)
\]
2. Technical Background: Dynamic Forces

Example Fourier Series

\[
F(t) = 185 \text{lbf} + 79.6 \text{lbf} \cdot \sin[2\pi(2 \text{Hz})(t) - 0] + \\
12.6 \text{lbf} \cdot \sin[2\pi(4 \text{Hz})(t) + \pi/2] + 10.1 \text{lbf} \cdot \sin[2\pi(6 \text{Hz})(t) - \pi] + \\
8.88 \text{lbf} \cdot \sin[2\pi(8 \text{Hz})(t) - \pi/2]
\]
2. Technical Background: Dynamic Forces

Example Fourier Series

- Review spectrum representation.

\[ F(t) = 185 \text{lbf} + 
79.6 \text{lbf} \cdot \sin[2\pi(2 \text{ Hz})(t) - 0] + 
12.6 \text{lbf} \cdot \sin[2\pi(4 \text{ Hz})(t) + \pi/2] + 
10.1 \text{lbf} \cdot \sin[2\pi(6 \text{ Hz})(t) - \pi] + 
8.88 \text{lbf} \cdot \sin[2\pi(8 \text{ Hz})(t) - \pi/2] \]

“Time Domain” or “Waveform”

“Frequency Domain” or “Spectrum”

Equivalent Representations
2. Technical Background: Dynamic Forces

- Specialize Fourier Series for Human Induced Forces

\[ F(t) = 0 + \sum_{h=1}^{4} Q \cdot \alpha_h \cdot \sin(2 \cdot \pi \cdot h \cdot f_{\text{step}} \cdot t - \phi_h) \]

- Therefore, need Q, DLFs, range of step frequencies, and phase lags.

- Don’t need or want the “DC Offset”
2. Technical Background: Dynamic Forces

- Sources of Fourier Series Parameters
  - Walking on Floors
    - British SCI P354 (Smith et al. 2007).
    - Davis and Murray (2010) AISC NASCC.
  - Rhythmic Activities (dancing, concerts, aerobics)
    - DG11.
    - SCI P354.
2. Technical Background: Dynamic Forces

- Sources of Fourier Series Parameters
  - Stair Ascents and Descents
  - Miscellaneous
2. Technical Background: Dynamic Forces

- Example Fourier Series Parameters
  - AISC DG11 Walking
    - Bodyweight $Q = 157$ lbf
    - $\alpha = 0.5, 0.2, 0.1, 0.05$
    - Phase lags not given
  - SCI P354 Walking
    - $Q = 168$ lbf
    - $\alpha = 0.46, 0.1, 0.08, 0.07 (+/-)$
    - $\phi = 0, -\pi/2, \pi, \pi/2$
2. Technical Background: Dynamic Forces

- Step Frequency
  - Depends on activity. Use walking for example.
  - $f_{Step}$ usually between 1.6 Hz and 2.2 Hz
  - Average is 1.9-2.0 Hz

Figure 2: Probability density function for (a) step frequency and (b) step length.

Source: Zivanovic et al. (2007) “Statistical characterisation of parameters defining human walking as observed on an indoor passerelle.”
2. Technical Background: Dynamic Forces

- **Relevance of Step Frequency Range**
  - Walking $f_{Step}$ must be between 1.6 Hz and 2.2 Hz.
  - Therefore $hf_{Step}$ in the Fourier Series are also bound.

$$F(t) = 0 + \sum_{h=1}^{4} Q \cdot \alpha_h \cdot \sin(2 \cdot \pi \cdot h \cdot f_{Step} \cdot t - \phi_h)$$

- The maximum walking harmonic frequency is about 9 Hz.
- If floor natural frequency is less than about 9 Hz, resonance is possible → “Low Frequency Floor”
2. Technical Background: Dynamic Forces

- Individual Footstep Pulses and Impulses
  - Used when resonance not possible. “High Frequency Floors.”
  - Used to compute response to one footstep.
  - Example from DG11 Chapter 6.

\[ F(t) / F_m = 1/2 \left[ 1 - \cos \left( \frac{t}{t_p} \right) \right] \]
Natural Modes of Vibration

If a structure is displaced, released, and allowed to move freely, the motion will:

- Be at a specific frequency → natural frequency
- Be in a specific structural shape → mode shape
- Decay at a specific rate → damping

Natural modes are properties of the structure.

Natural modes do not depend on the dynamic load.
2. Technical Background: Dynamic Properties of Floors

- Natural Modes
  - Structures have many natural modes.
  - The lowest frequency mode is referred to as the fundamental mode.
  - Some modes are more responsive than others. Quantify with the Frequency Response Function (FRF).
2. Technical Background: Dynamic Properties of Floors

- **FRF Definition**: ratio of steady state response to force input.

  
  ![Diagram](image)

  - 1 lbf sinusoidal load
  - Sinusoidal Force Input, Freq. = \( f \)
  - Sinusoidal Response, Freq. = \( f \)

  - Repeat for a range of \( f \) and plot the steady state acceleration response.

  - Steady state response amplitude, \( \%g \)

Equal load applied at each frequency, but the structure responded a LOT more at the natural frequencies.
2. Technical Background: Dynamic Properties of Floors

- Variables That Affect the Response
  - Damping (Response Inversely Proportional)
    - Depends mostly on nonstructural components.
    - Limited experimental data available.
    - Low end: 0.5% of critical damping for bare slabs.
    - High end: 5% of critical damping for floors with full height drywall partitions. 6% is often used when crowds are on the floor.
2. Technical Background: Dynamic Properties of Floors

- Variables That Affect the Response
  - Inversely proportional to effective mass (aka modal mass) which depends on the psf and extent of motion.

Measured 8.3 Hz Mode
Two bays with significant motion.
Larger effective mass.
Less Responsive.

Measured 8.9 Hz Mode
One bay with significant motion.
Smaller effective mass.
More Responsive.
Variables That Affect the Response
- Location on the floor. Proportional to mode shape amplitude.
2. Technical Background: Response Prediction

- Resonant Responses

\[ F(t) = \sum_{h=1}^{4} Q \cdot \alpha_h \cdot \sin(2 \cdot \pi \cdot h \cdot f_{\text{Step}} \cdot t - \phi_h) \]

Matches a Natural Frequency

Causes Resonance
2. Technical Background: Response Prediction

- Low Frequency vs High Frequency Floors
  - If fundamental frequency exceeds 9 Hz, then the fourth harmonic can’t reach it.

9 Hz

Resonance Possible
Low Frequency Floors (LFF)  Resonance Unlikely
High Frequency Floors (HFF)
2. Technical Background: Response Prediction

- **Low Frequency vs High Frequency Floors**
  - Maximum response for LFF is from resonant response.
  - Maximum response for HFF is from a single footstep.

![Graph: LFF With Resonant Build-Up](image1)

![Graph: HFF with Series of Individual Footstep Responses](image2)

- Peak Accel. = 1.18%g
- Max. 2 sec. RMS Accel. = 0.723%g

- Peak Accel. = 0.386%g
- Max. 2 sec. RMS Accel. = 0.136%g
2. Technical Background: Response Prediction

- Example Measured Resonant Response
  - Natural Frequency = 5.00 Hz. Walking at 1.67 Hz
  - Fourier series terms at 1.67 Hz, 3.33 Hz, 5.00 Hz, 6.67 Hz
    \[ F(t) = \sum \alpha_h Q \sin(2\pi h f_{Step} t - \phi_h) \]
  - Conclusion: vast majority of response is due to one harmonic exciting one mode.

\[ \text{Response to 1}\text{st Harmonic (@ 1.67 Hz)} \]
\[ \text{Response to 2}\text{nd Harmonic (@ 3.33 Hz)} \]
\[ \text{Response to 3}\text{rd Harmonic (5.00 Hz)} \]
\[ \text{Response to 4}\text{th Harmonic (6.67 Hz)} \]

Peak Accel. = 1.36%g
RMS Accel. = 0.531%g

\[ f_{Step} \]
2. Technical Background: Response Prediction

- Resonant Response Prediction Options
  - Single Degree of Freedom (SDOF) Idealization.
    - Closed form solutions.
    - Practical for manual calculations.
  - Finite Element Analysis.
    - Response History Analysis (aka Time History).
    - Useful for cases with unusual requirements.
2. Technical Background: Response Prediction

- Resonant Response Prediction Options
  - SDOF Idealization

![Diagram of a mass-spring-damper system with the equation $f_n \propto \sqrt{\frac{K}{M}}$ (Natural Frequency) and $\beta = \frac{C}{2(2\pi f_n)M}$ (Modal Damping Ratio).]

Driving frequency matches natural frequency.

$P\sin(2\pi f_n t)$

$a_{\text{Steady State}} = \frac{P}{2\beta M}$

The basis for almost all modern SDOF approximate methods for low frequency floors.
2. Technical Background: Response Prediction

- Resonant Response Prediction Options
- SDOF Idealization
  - Specialize for low frequency floor vibrations.
    - \( P \) = the Fourier series term amplitude that causes resonance. Recall: \( F(t) = \sum \alpha_h Q \sin(2 \cdot \pi \cdot h \cdot f_{Step} \cdot t - \phi_h) \)
    - \( M \) = an estimate of the effective mass.
    - Also need to account for incomplete resonant build-up.

\[ a_{\text{SteadyState}} = \frac{P}{2\beta M} \]

Predicted acceleration due to human activity at resonance.
2. Technical Background: Response Prediction

“Off Resonance” Response of SDOF System

We’re sometimes interested in the steady-state response of the system to a sinusoid with amplitude $P$ and $f \neq f_n$.

$$a_{\text{Steady State}}(f_n) = \frac{P}{2\beta M}$$

$$a_{\text{Steady State}}(f) = \frac{P / M}{\sqrt{\left(\frac{f_n}{f}\right)^2 - 1}^2 + \left(2\beta \frac{f_n}{f}\right)^2}$$
2. Technical Background: Human Tolerance

- Depends on the setting, what they’re doing, and the frequency of vibration.
- Most sensitive in 4 Hz to 8 Hz band.
- 0.5%g is the limit for quiet spaces.

DG11 Figure 2.1 (Murray et al. 1997)
2. Technical Background: Equipment Tolerance

- Equipment limits much more stringent than human comfort limits.
- Manufacturers often provide limit in the form of acceleration spectrum.

1. Shape of the spectrum shows why HFF preferred for sensitive equipment areas.
2. In the absence of a specific manufacturer limit, the AISC DG11 Chapter 6 “generic limits” can be used.
3. DG11 Chapter 4 Introduction

- Chapter 4 applies to the usual cases.
  - Dynamic force from human walking.
  - Human comfort is the issue. Legitimate complaints constitute failure.

- Organization of this section.
  - Introduction to the criterion.
  - 3.1 Acceleration prediction equation—derivation.
  - 3.2 Acceleration prediction equation—implementation.
  - 3.3 Acceleration tolerance limits for human comfort.
  - 3.4 Example.
  - 3.5 Accuracy.
3. DG11 Chapter 4 Introduction

- Criterion in a nutshell
  - Under no circumstance can the natural frequency be less than 3.0 Hz \( \rightarrow \) vandal jumping.
  - Predict the peak acceleration due to walking. Compare that to the human tolerance.

\[
a_p = \frac{P_0 e^{-0.35f_n}}{\beta W} \leq a_o
\]

- Predicted Peak Acceleration, g
- Human tolerance limit, g
3.1 DG11 Ch. 4 Acceleration Prediction Equation—Derivation

- Derivation useful to avoid incorrectly applying the equation.
- Start with the SDOF steady-state acceleration equation.

\[ a(t) = P \cdot \sin(2\pi f_n t) \]

(at resonance)

\[ a_{\text{SteadyState}} = \frac{P}{2\beta M} \]
3.1 DG11 Ch. 4 Acceleration Prediction Equation—Derivation

- Specialize by defining $P$ for walking.

\[
\text{Harmonic} \quad \text{DLF} \quad P = \text{Amplitude} = \text{DLF} \times \text{Bodyweight} \quad (\text{lbf})
\]

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>DLF</th>
<th>$P$ = Amplitude = DLF x Bodyweight (lbf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>78.5</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>31.4</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>15.7</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>7.85</td>
</tr>
</tbody>
</table>

Assumed Bodyweight = 157 lbf

Could select $P$ from this table, but Allen and Murray found a more elegant method.
Specialize by defining $P$ for walking

Write an equation that works regardless of which harmonic is applicable.

\[ DLF = \alpha = 0.83e^{-0.35f_n} \]

\[ P = Q\alpha = (Q)(0.83)e^{-0.35f_n} \]

Practical Application: Engineers sometime attempt to apply DG11 Ch. 4 to running, stair descents, etc. The DLF being used here are not similar to those for other types of dynamic forces.
3.1 DG11 Ch. 4 Acceleration Prediction Equation—Derivation

- Adjust the equation to account for
  - Incomplete resonant build-up.
  - Walker and annoyed person are not at the same location at the same time.
  - Use a reduction factor, $R = 0.5$ for floors with two-way mode shapes or $R = 0.7$ for footbridges.
Effective Mass

- Estimate the weight of plan area participating in the motion. Call that $W$.
- Effective mass is $0.5W$ because the mode shape is similar to a beam (Allen and Murray 1993).
3.1 DG11 Ch. 4 Acceleration Prediction Equation—Derivation

Putting it all together.

\[ a_{\text{SteadyState}} = \frac{P}{2\beta M} \]

Call this \( P_o \)

\[ a_p = \frac{R_P}{2\beta M} = \frac{RQ0.83e^{-0.35f_n}}{2\beta 0.5W/g} = \frac{RQ0.83e^{-0.35f_n}}{\beta W} g = \frac{P_o e^{-0.35f_n}}{\beta W} g \]

\( P_o = (0.5)(157 \text{ lbf})(0.83) = 65 \text{ lbf for floors} \)

\( P_o = (0.7)(157 \text{ lbf})(0.83) = 92 \text{ lbf for footbridges} \)

\[ a_p = \frac{P_o e^{-0.35f_n}}{\beta W} g \]  
DG11 Eq. 4.1.
Observations

- Natural frequency is only used to set the harmonic load.

\[
ap = \frac{R_P}{2\beta M} = \frac{RQ0.83e^{-0.35f_n}}{2\beta 0.5W / g} = ...
\]

- \(P_o\) is in lbf, but is a combination of parameters. One can’t say “the floor is loaded with a 65 lbf point load.”
Natural Frequency

- Approach: Compute $f_{n,b}$ for the beam, $f_{n,g}$ for the girder, and combine to get the bay’s frequency, $f_n$.

- Natural frequency in Hz for a simply supported beam with uniform mass and $EI$.

$$f_{n,b} = \frac{\pi}{2} \sqrt{\frac{E \cdot I}{m \cdot L^4}}$$

What mass is used? Use the absolute best estimate. It’s not conservative to over- or under-estimate the mass. (over-estimation will be conservative for $f_n$, but unconservative for $W$.)
Another one of these “don’t be misled by…” items: \( m \) is mass \textbf{attached to} the beam, plus the beam itself. Don’t think of it as load. (This will clear up questions such as “Does \( m \) include precomposite DL?”)

\[
f_{n,b} = \frac{\pi}{2} \sqrt{\frac{E \cdot I}{m \cdot L^4}}
\]
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- **Natural Frequency**
  - Re-write the equation in a form that’s easier to remember. $f_{n,b}$ in Hz.
  - Note how $w$ creeps in. We’re still dealing with mass, not load. $w$ is just part of $m=w/g$.

$$f_{n,b} = \frac{\pi}{2} \sqrt{\frac{E \cdot I}{m \cdot L^4}} = \frac{\pi}{2} \sqrt{\frac{g \cdot E \cdot I}{w \cdot L^4}} = \frac{\pi}{2} \sqrt{\frac{g \cdot 384 \cdot E \cdot I}{5 \cdot w \cdot L^4}}$$

$$= \frac{\pi}{2} \sqrt{\frac{5}{384 \sqrt{g \cdot \frac{384 \cdot E \cdot I}{5 \cdot w \cdot L^4}}} = 0.179, \sqrt{\frac{g}{\Delta}} \approx 0.18, \sqrt{\frac{g}{\Delta}}$$

\[\Delta = \frac{5wL^4}{384EI} \text{ (in.)}\]

$g = 386 \text{ in./s}^2$
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

Natural Frequency

- Girder natural frequency computed using the same equation. Assume uniform mass over the tributary width.

\[ f_{n,g} = \frac{\pi}{2} \sqrt{\frac{E \cdot I}{m \cdot L^4}} = 0.179 \sqrt{\frac{g}{\Delta}} \approx 0.18 \sqrt{\frac{g}{\Delta}} \]
Natural Frequency

- There are usually two girders. Use the one with the lowest natural frequency. Discard the other one.
- Spandrel girders supporting cladding are assumed to be fixed. Modal test data indicates significant restraint even if cladding has vertical slip connections.
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Natural Frequency
  - Bay natural frequency, $f_n$, computed as the combination of $f_{n,b}$ and $f_{n,g}$. Use Dunkerley’s Equation.
    \[
    \frac{1}{f_n^2} = \frac{1}{f_{n,b}^2} + \frac{1}{f_{n,g}^2} \quad \text{(DG11 Eq. 3.2)}
    \]
  - Similar to before,
    \[
    f_n \approx 0.179 \sqrt{\frac{g}{\Delta_b + \Delta_g}} \approx 0.18 \sqrt{\frac{g}{\Delta_b + \Delta_g}} \quad \text{(DG11 Eq. 3.4)}
    \]
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Natural Frequency
  - Use fully composite transformed MOI.
    - Amplitude of horizontal shear between beam and deck is very small, so won’t overcome deck puddle welds or friction.
    - Exceptions: noncomposite if actual physical separation between the beam and deck.
    - Effective slab width a little different from the AISC Spec. Ch. I value. See DG11 Section 3.2.

\[
b_{e, Each Side} = \min \left( \frac{1}{2} \text{Beam Spacing}, 0.2L_b \right)
\]

Edge Distance
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Natural Frequency
- Moment of Inertia
  - Use dynamic elastic modulus for concrete.
  - Same as in the AISC Specification Ch. I except 35% higher. DG11 Section 3.2.

\[
E_c = 1.35w^{1.5}\sqrt{f'_c}
\]

- Dynamic elastic modulus, ksi
- Compressive strength, ksi
- Unit weight, pcf
Natural Frequency

- Modified MOI for open-web steel joists.
  - These have significant shear deformation (unlike w-shapes) and web eccentricities.

![Diagram of 2 in. +/- (Beavers 1998)]
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Natural Frequency
  - Modified MOI for open-web steel joists, continued.

\[ I_{\text{eff}} = \frac{1}{\gamma} + \frac{1}{I_{\text{chords}}} \]  
(DG11 Eq. 3.18)

\[ \gamma = \frac{1}{C_r} - 1 \]  
(DG11 Eq. 3.19)

\[ C_r = 0.90 \left(1 - e^{-0.28(L/D)}\right)^{2.8} \text{ where } 6 \leq L / D \leq 24 \]  
(DG11 Eq. 3.16)

\[ C_r = 0.721 + 0.00725(L / D) \text{ where } 10 \leq L / D \leq 24 \]  
(DG11 Eq. 3.17)
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Natural Frequency
  - Modified MOI for open-web steel joists, continued.
    - Use $I_{eff}$ to compute $f_{n,b}$.

$$f_{n,b} = \frac{\pi}{2} \sqrt{\frac{E \cdot I_{eff}}{m \cdot L^4}} = \frac{\pi}{2} \sqrt{\frac{g \cdot E \cdot I_{eff}}{w \cdot L^4}} = 0.179 \sqrt{\frac{g}{\Delta}} \approx 0.18 \sqrt{\frac{g}{\Delta}}$$

Computed with $I_{eff}$
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Natural Frequency
  - Modified MOI for joist girders.
    - Slab is physically separated from the top chord.
    - Joist seats provide limited horizontal shear stiffness between the joist girder and the slab.

![Diagram of joist girders and slab with labels: Slab, Joist Seat (Deforms Elastically), Joist Girder Top Chord]
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Natural Frequency
  - Modified MOI for joist girders.

\[ I_{\text{eff}} = I_{nc} + \frac{I_{\text{comp}} - I_{nc}}{4} \]  

(DG11 Eq. 3.14)

where

\[ I_{nc} = C_r I_{\text{chords}} \]  

(DG11 Eq. 3.15)

\[ C_r = 0.90 \left(1 - e^{-0.28(L/D)}\right)^{2.8} \text{ where } 6 \leq L / D \leq 24 \]  

(DG11 Eq. 3.16)

\[ \gamma = \frac{1}{C_r} - 1 \]  

(DG11 Eq. 3.19)
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- **Effective Weight**
  - Recall the acceleration prediction equation.
  
  \[
  a_p = \frac{RP}{2\beta M} = \frac{RQ0.83e^{-0.35f_n}}{2\beta 0.5W/g} = \frac{P_0e^{-0.35f_n}}{\beta W} g
  \]
  
  - **Modal Mass or Effective Mass**
  - **Effective Weight** (DG11’s term)

- **W** is double the effective mass.
- Compute as a combination of “beam mode” and “girder mode” effective weights.
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Effective Weight
  - Beam Mode Effective Weight
    - Beams bend, girders rigid.
    - $B_b$ (perpendicular to beams) depends on ratio of slab stiffness to beam stiffness.

- Very flexible slab $\rightarrow$ very small $B_b$.
- Very stiff slab $\rightarrow$ very large $B_b$.
- Reality: varies, but it’s often a little larger than the girder length.
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Effective Weight
  - Beam Mode Effective Weight

\[ W_b = p_b \cdot B_b \cdot L_b \]  
(DG11 Eq. 4.2, slightly modified)

where

- \( p_b = \text{floor weight, psf} \)
- \( L_b = \text{beam length, ft} \)
- \( B_b = \min \left| C_b \left( \frac{D_s}{D_b} \right)^{0.25} L_b \right| \)  
  (2/3 the "floor width", ft)  
  (DG11 Eq. 4.3a)

Include the masses that are moving during the beam bending mode. In other words: LL, deck, slab, SDL, and beams. Not girders or columns.
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Effective Weight
  - Beam Mode Effective Weight
    - $C_b$ Determination. Default is 2.0. Other option is 1.0 at a free edge “mezzanine condition.”

$C_b = 1.0$ for these bays because beam parallel to free edge.

Interior Opening (Atrium)
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Effective Weight
  - Beam Mode Effective Weight
    - Floor width: distance perpendicular to the beams to discontinuity in the framing such as slab edge, interior opening, or framing direction change.
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Effective Weight
  - Beam Mode Effective Weight
    - $D_s$ and $D_b$ are slab and beam transformed flexural stiffness, respectively, per unit width. See DG11 Section 4.2. Units: in.\(^4/\)ft

$$D_s = \frac{(12 \text{ in.})d_e^3}{12n}$$

$$D_b = \frac{I_{beam}}{\text{Beam Spacing}}$$

- Fully composite transformed MOI for W-shapes.
- Effective MOI for open-web joists.
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Effective Weight
  - Beam Mode Effective Weight
    - Continuity Effect. See DG11, bottom, left of Page 18. Increase $W_b$ by 50% if both of the following are true. (Similar liberalization applies to girders.)
      - At least one adjacent beam span is 70% of $L_b$.
      - Beams connect to girder webs (not joist seats).
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Effective Weight
  - Girder Mode Effective Weight
    - Beams rigid, girders bend.
    - Extent of the motion perpendicular to girders relates to ratio of beam to girder stiffness.
      - Very flexible beams $\rightarrow$ small $B_g$.
      - Very stiff beams $\rightarrow$ large $B_g$.
      - Reality: varies, but often a little larger to perhaps double the beam length.
    - Calcs almost identical to beam bending mode.
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

Effective Weight

Girder Mode Effective Weight

\[ W_g = p_g \cdot B_g \cdot L_g \]  
(DG11 Eq. 4.2, slightly modified)

where

- \( p_g = \text{floor weight, psf} \)
- \( L_g = \text{girder length, ft} \)
- \( B_g = \min \left( \left. \frac{C_g \left( \frac{D_b}{D_g} \right)^{0.25} L_g}{2/3 \text{ the "floor length"}} \right\} , \text{ft} \)  
  (DG11 Eq. 4.3b)

Include the masses that are moving during the girder bending mode. In other words: all of the floor system.
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Effective Weight
  - Girder Mode Effective Weight
    - $C_g$ depends on type of connection to girder.
      - 1.6 if joist seats.
      - 1.8 if typical beam-to-girder connections.
    - Floor length: same idea as floor width except in the other direction. Floor length is perpendicular to the girders.
    - Continuity effect: same as for beams except only applies if the girder runs over the column. Stacked columns, in other words.
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Effective Weight
  - Combined Beam and Girder Mode Effective Weight
    - Use weighted average.
    - Deflections from the natural frequency calculations.

\[ W = \frac{\Delta_b}{\Delta_b + \Delta_g} W_b + \frac{\Delta_g}{\Delta_b + \Delta_g} W_g \]  
(DG11 Eq. 4.4)
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Damping
  - Estimate from DG11 Table 4.1, or
- Cumulative
  - Structural system: 0.01
  - Ceiling and ductwork: 0.02
  - Paper office: 0.01
  - Electronic office: 0.005
  - Churches, schools, malls: 0.0
  - Drywall partitions: 0.05
3.2 DG11 Ch. 4 Acceleration Prediction Equation—Implementation

- Mass (Typical Values)
  - Member self weight
  - Slab and deck self weight (nominal)
  - Superimposed dead load: 4 psf
  - Paper office live load: 11 psf
  - Electronic office live load: 8 psf
  - Residence live load: 6 psf
  - Assembly or mall live load: 0 psf
3.3 DG11 Ch. 4 Acceleration Limits for Human Comfort

- DG11 Fig. 2.1 and Table 4.1.

DG11 authors flatten out the curve and ignore the benefit outside the 4 Hz to 8 Hz range.
References


